Quick Refresher on The Chain Rule of Differentiation

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1 Introduction

This article is designed as simple refresher on the chain rule of differentiation. We will explore two simple practice problems. Comfort using the chain rule is crucial when we manually derive the first partial derivative of the Black-Scholes call option with respect to stock price and show that it is in fact delta. Also be aware that many questions like these often find there way to interview questions for quant finance roles.

If we let $g(.) = g(f(x))$ then the chain rule can be written as:

$$\frac{dg(x)}{dx} = \frac{dg(x)}{df(x)} \cdot \frac{df(x)}{dx} \tag{1}$$

Let us now see why this is helpful with two simple practice problems.

2 Practice Problems

1. Find $\frac{df(x)}{dx}$ where $f(x) = x^x$

We let $g(x) = ln(f(x)) = xlnx$ and then find:

$$\frac{dg(x)}{dx} = lnx + \frac{1}{x} \cdot x = lnx + 1$$

$$\frac{dg(x)}{df(x)} = \frac{1}{f(x)}$$

Now we simply apply the chain rule:

$$\frac{dg(x)}{dx} = \frac{dg(x)}{df(x)} \cdot \frac{df(x)}{dx}$$

$$lnx + 1 = \frac{1}{f(x)} + \frac{df(x)}{dx}$$

$$\frac{df(x)}{dx} = f(x)[lnx + 1]$$

$$\frac{df(x)}{dx} = x^x[lnx + 1]$$

2. Find $\frac{df(x)}{dx}$ where *f(x)* $= x^{e^x}$

Again, we let *g(x)* $= ln(f(x)) = e^x \cdot lnx$ and then find:

$$\frac{dg(x)}{dx} = e^x \cdot lnx + \frac{e^x}{x}$$

$$\frac{dg(x)}{df(x)} = \frac{1}{f(x)}$$

Apply the chain rule:

$$\frac{dg(x)}{dx} = \frac{dg(x)}{df(x)} \cdot \frac{df(x)}{dx}$$

$$e^x \cdot lnx + \frac{e^x}{x} = \frac{1}{f(x)} \cdot \frac{df(x)}{dx}$$

$$\frac{df(x)}{dx} = f(x)[e^x \cdot lnx + \frac{e^x}{x}]$$

$$\frac{df(x)}{dx} = x^{e^x}[e^x \cdot lnx + \frac{e^x}{x}]$$